ERRORS IN ALGEBRAIC STATEMENTS TRANSLATION DURING THE CREATION OF AN ALGEBRAIC DOMINO

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We present a research study which main objective is to inquire into secondary school students´ ability to translate and relate algebraic statements which are presented in the symbolic and verbal representation systems. Data collection was performed with 26 14-15 years old students to whom we proposed the creation of an algebraic domino, designed for this research, and its subsequent use in a tournament. Here we present an analysis of the errors made in such translations. Among the obtained results, we note that the students found easier to translate statements from the symbolic to the verbal representation and that most errors in translating from verbal to symbolic expressions where derived from the particular characteristics of algebraic language. Other types of errors are also identified.
KEYWORDS: Algebraic language, domino, errors, translation between representation systems, verbal representation.

INTRODUCTION

At the end of secondary education, students still experience difficulties when relating verbal and symbolic representations of algebraic statements despite having used algebraic symbolism in the previous courses. This fact constitutes a difficulty for solving problems that requires the use of algebraic language. Knowing errors that students make when translating statements between the verbal and symbolic representation systems can help in the study of algebraic symbolism acquisition as well as of algebraic problem solving.

Various researchers (Arcavi, 1994; Bednarz, Kieran, & Lee, 1996; Kaput, 2000 Palarea, 1998; Ruano, Socas & Palarea, 2008) have highlighted the existing difficulties in the acquisition of an appropriate command and understanding of the algebraic language and have identified some of the most frequent errors in tasks such as generalizing, modelling and solving equations. For example they identify errors related to the need for closure, the particularization of expressions, the inappropriate use of brackets and the confusion between the operations multiplication and power. Some other authors have explored the role of verbal writing in algebra learning (MacGregor, 1990; Wollman, 1983). However, we haven't found much information on secondary students' performance on translation of statements between verbal and symbolic representations. The analysis of translation processes (in both ways) can help to (a) deepen in the students´ understanding of symbolic language and (b) identify difficulties that pupils come across with when writing verbal statements in a symbolic form.

In this study we focus on external representations, which are the noticeable ones in the students' performance. We considered that a representation system is a group of symbols,

graphs and rules allowing the representation of a mathematical structure. It is also characterized by following some certain systematization (Castro & Castro, 1997; Rico, 1997).

AIMS

The general aim of the study is *analyzing secondary students' process of translating general statements of numeric relations (algebraic statements, from now on) between the verbal and symbolic representation systems.* This aim leads us to establish three specific goals: (a) create an instrument allowing the exploration of the translation process between the symbolic and verbal representation systems, as we did not find such tool in the specialized literature consulted by us; (b) analyze and classify students' errors on such translations; and (c) describe the relations that students establish between verbal and symbolic representations of the same algebraic statement.

This paper focus on the first two aims (for further detail see Rodríguez-Domingo, 2011). We understand errors as results of unsuitable cognitive schemes (Matz, 1980; Socas, 1997) and, following Rico's (1995) claim, we consider errors as a permanent possibility for the acquisition and consolidation of knowledge regardless the fact that they tend to have negative implications at school.

METHODOLOGICAL FRAMEWORK

We selected an intentional sample of 26 14-15 years old students. The socio-cultural and academic levels of the students were low because the school is located in a conflictive neighbourhood. The pupils' interest towards learning and their class attendance was also quite low. Concerning the students' academic situation, six of them were repeating the 4th year of secondary education, and most of the remaining 20 pupils had failed mathematics in previous years. Before the data collection, these students had studied the arithmetic and algebra sections of the secondary curriculum corresponding to the last year of compulsory education which includes translating algebraic statements and formulating and solving equations and inequalities.

Data collection

The particular students' characteristics led to the design of a data collection instrument aiming to raise their interest. We used an instrument based on the game domino, which pupils were quite familiar with. According to authors such as De Guzmán (1984) and Moyles (1990), any material presented as a game improves the learning process and encourages motivation among the students.

The teacher of these students (first author of this paper) performed the data collection in two different phases: (1) Students had to build the dominoes pieces individually in the first phase, translating some statements from verbal to symbolic representations, and *vice versa*; and (2) Students played a tournament using the complete and correct algebraic domino. This paper focuses on the first phase.

During the first phase, our aim was analyzing the students' errors when translating algebraic statements between the verbal and symbolic representation systems. The revision of the students' previous work on the topic, based on the analysis of the students' textbook and the activities worked in class, led to identifying the numerical relations that students have previously encountered in algebraic activities. They were related to number addition and subtraction, multiplication, division, powers, square roots, consecutive numbers, and even and odd numbers. Taking these relations into account, we designed 12 statements using only natural numbers. The kind of numerical relation constitutes the first task variable.

Six statements were presented verbally and the other six symbolically. Considering the possible combinations among the identified numerical relations, we decided that among the six statements of each type, one of them would be just additive, another only multiplicative, one just involving powers, and the remaining three would deal with the combinations of the previous relations (additive-multiplicative, additive-power, multiplicative-power). We also decided to design an equal number of open and closed1 statements as well as of sequential2 and non sequential verbally expressed statements.

Table 1 shows the definitive designed statements and its main characteristics according to the task variables considered. The numeration allows us to identify each statement in the data analysis.

Table 1. Statements to be translated during the first phase

Statements	Closed	Sequential	Relations involved
Verbal representation			
1) The product of one number's half multiplied by another number's triple	No	No	Multiplicative
3) A certain number plus its consecutive one, equals another number minus two	Yes	Yes	Additive
4) One number multiplied by the square of that number, equals the cube of that same number.	Yes	Yes	Multiplicative and power
7) The square of the addition of two consecutive numbers	No	No	Additive and power

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[1] We use the label "closed", in opposition to "open", to refer to those statements that establish equality among statements, that is, the one equivalent to an equation.

[2] The denomination "sequential" distinct verbally expressed statements that can be translated to a symbolic representation by strictly following from left to right the order in which the terms and relations within the statement are mentioned.

Statements	Closed	Sequential	Relations involved
8) One even number minus the quarter of another number	No	Yes	Additive and multiplicative
11) The square of a number's square root equals that same number	Yes	No	Power
Symbolic representation			
2) $4 \cdot \left(\dfrac{x}{2} \right) = 2x$	Yes		Multiplicative
5) $x \cdot (x+1) = 7x$	Yes		Additive and multiplicative
6) $(x \cdot y)^3$	No		Multiplicative and power
9) $\left(\sqrt{x} \right)^y$	No		Power
10) $x^2 - y^2 = 11$	Yes		Additive and power
12) $x + (x+1) - 4$	No		Additive

We used these statements to build incomplete dominoes forming an already finished game (see Figure 1). Students were asked to fill in the gaps so that the matching of dominoes was appropriate. They were warned that there were no blank parts unlike the regular domino game. The students completed the dominoes individually and wrote down their names in the worksheet.
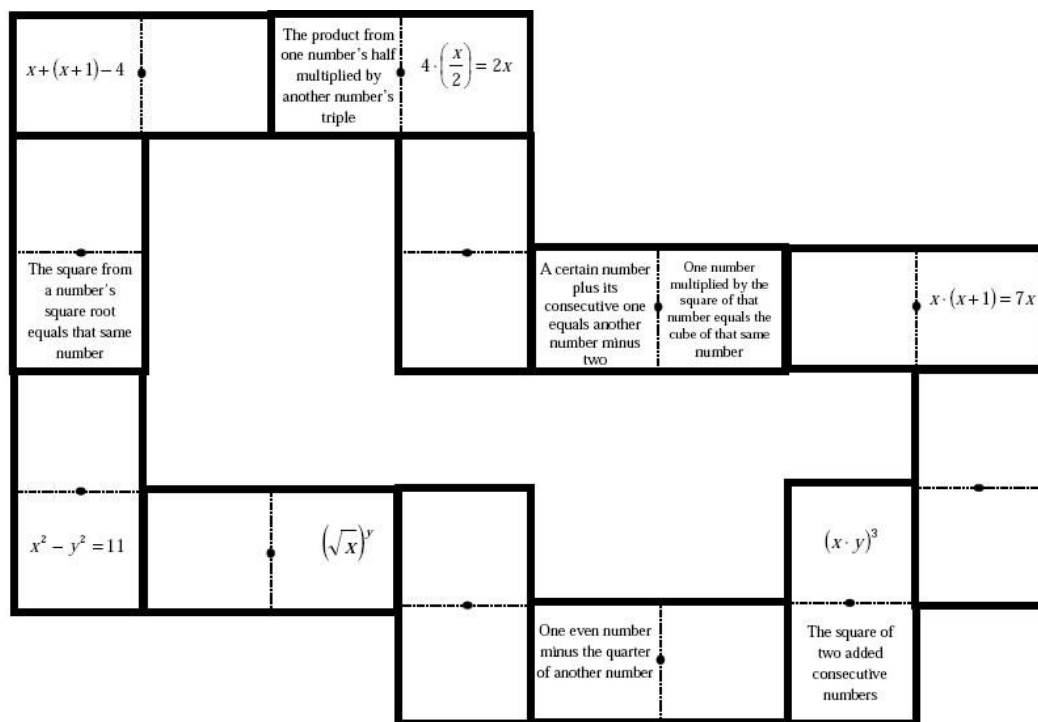
Figure 1. Document for the first phase within the data collection process

DATA ANALYSIS

We focussed our analysis on the type of translation that students performed in each case as well as the type of error made, their frequency and the statement where they were identified. Firstly, we present the number of different errors detected in each statement in both types of translations (see Table 2).

Table 2. Number of different errors committed during statement translation

From symbolic to verbal	Number of errors	From verbal to symbolic	Number of errors
Statement 2	1	Statement 1	13
Statement 5	4	Statement 3	7
Statement 6	2	Statement 4	1
Statement 9	4	Statement 7	8
Statement 10	2	Statement 8	14
Statement 12	2	Statement 11	0
Total	15	Total	43

In order to classify the errors, we developed a categorization adapting the one of Socas (1997) to our data. Table 3 shows the final categories we adopted and the codes assigned to them. Three major types of errors where distinguished: (a) upon the statement's completeness, (b) derived from arithmetic, and (c) derived from the peculiar features of algebraic language.

Table 3. Errors classification

Category	Subcategory	Code
I. Upon the statement's completeness	Incomplete	I.1
	Overabundant	I.2
II. Derived from arithmetic	Brackets	II.1
	Fraction – product	II.2
	Power – product	II.3
	Addition – product	II.4
	Fraction – power	II.5
III. Derived from the peculiar features of algebraic language	Generalization	III.1
	Particularization	III.2
	Variables	III.3
	Structural complication	III.4

The first group makes reference to errors related to statements where there is some missing or extra symbol/word so that the expression is correct (incomplete versus overabundant). The category "derived from the arithmetic" includes errors originated by the incorrect interpretation of symbols, operations or the relations among them. Five subcategories are considered: "brackets" (II.1) refers to errors due to a misplaced or missing bracket; the other four subcategories deal with errors where the operations mentioned in their names are confused one by the other. For instance, students who expressed "the square root of a number multiplied by another one" as translation of the statement $\left(\sqrt{x}\right)^{y}$ incurred in a Power – product error (II.3)

Category III concerns the errors derived from the peculiar features of the algebraic language. This category includes errors inherent to the use of algebraic symbolism and it is composed by four subcategories:

- *Generalization*: an element or part of the statement which is a particular case is replaced by a more general one. For example, when the subtraction of 4 in the expression $x+(x+1)-4$ is verbally stated as "an even number is subtracted".
- *Particularization*: part of the statement with a general meaning is replaced by a particular case. For instance, when students translated symbolically "one even number minus the quarter of another number" as $2-(x/4)$.
- *Variable[3]*: lack of or unneeded distinction of variables/unknown that appear in the statement. An example is when students represented two different variables using the same symbol in the verbal statement "a certain number plus its consecutive one equals another number minus two".
- *Structural complication*: inappropriately interpretation of the algebraic statement's structure or part of it. For example, a student representing the statement "the square of two added consecutive numbers" as $x+(x+1)=x^2$.

Using this categorization, Table 4 shows a summary of the type of errors students incurred when translating statements from symbolic to verbal form and vice versa. It also includes the error frequencies, as well as the statement where the errors were found and the students who made them.

Table 4. Type of errors and frequency in the verbal to symbolic transformation

Type of error	Frequency	Statement	Students
From symbolic to verbal			
I.1	3	5	3B, 6B
		10	9B
I.2	1	5	3B
II.3	7	6	4B, 9C
		9	6B, 9B, 2C, 9C
		10	4B
III.1	4	2	4B
		5	4A
		10	4B

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[3] Due to the features of the statements employed in this study, it is not being distinguished if the used letter plays the role of variable or unknown factor.

Type of error	Frequency	Statement	Students
		12	4B
III.3	2	5	9B
		12	8B

From verbal to symbolic			
I.1	5	7	3A, 3B, 5B, 6B, 1C
I.2	4	1	5A, 4B, 6B, 3C
II.1	2	7	3A, 1C
II.2	2	1	5B
		8	2A
II.3	4	1	7A, 2B, 3B, 7C
II.4	1	4	7B
II.5	1	8	5A
III.2	7	8	2A, 5A, 4B, 5B, 6B, 6C, 7C
III.3	13	1	7A, 3B, 6B, 7B, 8B, 9B
		3	5A, 7A, 6B, 8B, 9B
		8	6A, 7A
III.4	15	3	8A, 6C
		7	2A, 6A, 3B, 5B, 6B, 1C, 9C
		8	4A, 7A, 8B, 9B, 1C, 5C

Discussion

One of the main results obtained is the higher frequency of errors (75%) originated in the translation from the verbal to the symbolic representation system. Twenty-one of the 23 students made errors when translating from verbal to symbolic while 8 made errors in the other direction. The verbal statements 1 and 8 presented the higher number of errors when translating to the symbolic system, followed by the statements 7 and 3. Symbolic statements 5 and 9 caused the most difficulties when translating to verbal representation. The data

collected do not allow us to identify the characteristics of these statements which caused this higher number of errors.

When translating from verbal to symbolic representation, most errors (65%) were due to the peculiar features of the algebraic language: variables and structural complication errors were the most frequent ones. It is noticeable the fact that errors due to particularization (III.2) were only produced in statement number 8 as the students use a particular even number to express the relation symbolically. The errors related to variables (III.4) were found in statements 3, 7, and 8. In statement 3, errors arose from pupils being mistaken in expressing two consecutive numbers symbolically. For example, student 8A expressed the addition of two consecutive numbers as $x + 1x$ while student 6C did it as $x + x + 2$. Regarding statement 7, errors emerged from changing the order of the stated operations (i.e., expressing the square of the addition as the addition of squared numbers). The structural complication errors found in statement 8 came up from interpreting incorrectly "any given even number", as x^2 or $x + 2$. The generalization error was the only one detected only in the verbal to symbolic translations.

When translating from symbolic to verbal presentation, the most frequent errors (41%) were due to confusion of the power and product operations (type II.3). Nevertheless, they did not make any errors referred to confusion involving other operations (II.2, II.3, II.4 & II.5). Among the error categories derived from the peculiarities of the algebraic language (the second most common error), students incurred in generalization errors (III.1) and variable errors (III.3). Some other errors corresponded to the incomplete and overabundant categories.

CONCLUSIONS

The designed instrument was useful for classifying the students' errors when translating written algebraic statements, as we aimed. Starting from previous studies and from the answers provided by the students, a categorization has been created in order to conduct an analysis of errors when translating statements both ways. It was concluded that most errors were incurred when transferring from verbal to symbolic representation. Differences in the type of errors committed were also identified. All this analysis process was useful as a first step to look into the students' ability to carry out translations and their comprehension of statements in each one of the representation systems abovementioned. Further exploration is needed to understand the causes of these errors and to find useful approaches to help students overcome them.

Acknowledgments

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