



## MEANINGS OF SPECIFIC TERMS RELATED TO THE CONCEPT OF FINITE LIMIT OF A FUNCTION AT ONE POINT

J.A. Fernández-Plaza, J.F. Ruiz-Hidalgo and L. Rico

Department of Didactics of Mathematics  
University of Granada-Spain

37<sup>th</sup> Conference of the International Group for the  
Psychology of Mathematics Education, Kiel, Germany  
July-August, 2013

# Contents

- 1 Problem
  - General idea
  - Goal
- 2 Background
- 3 Method
  - Subjects
  - Instrument
- 4 Results
  - Analysis of discussion
- 5 Conclusions

# General idea

## Specific terms

“to tend”  
“to approach”  
“to reach”  
“to exceed”  
“To converge”  
“to limit”

# General idea

Exploratory  
study



## Specific terms

“to tend”  
“to approach”  
“to reach”  
“to exceed”  
“To converge”  
“to limit”

“A limit is a number or point which a function cannot exceed” True or false? Explain your answer

# General idea

Exploratory  
study

## Specific terms

“to tend”  
“to approach”  
“to reach”  
“to exceed”  
“To converge”  
“to limit”

“A limit is a number or point which a function cannot exceed” True or false? Explain your answer

A student  
gives an  
answer

“It is true, because the limit is a point a function **gets close infinitely** but never **touches**”

# General idea

Exploratory  
study

## Specific terms

“to tend”  
“to approach”  
“to reach”  
“to exceed”  
“To converge”  
“to limit”

“A limit is a number or point which a function cannot exceed” True or false? Explain your answer

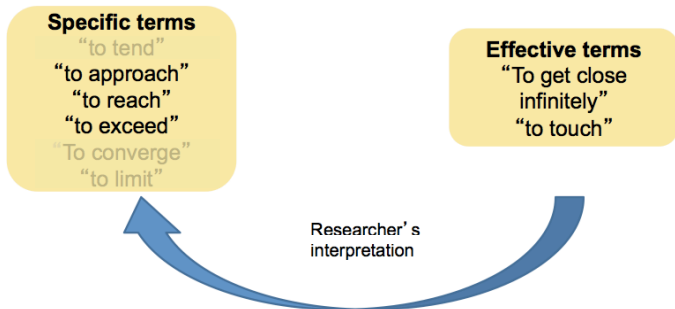
A student  
gives an  
answer

“It is true, because the limit is a point a function **gets close infinitely** but never **touches**”

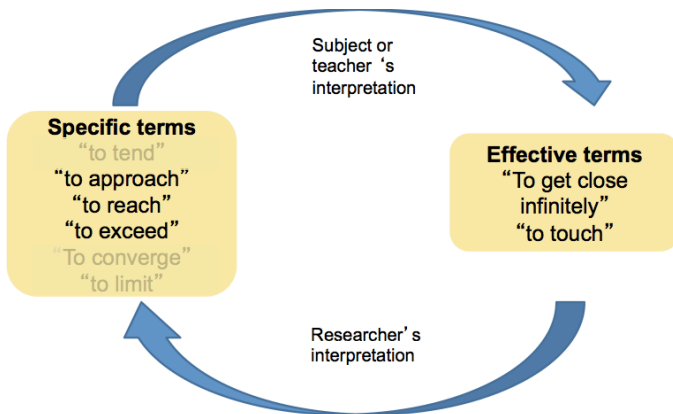
## Effective terms

“To get close infinitely”  
“to touch”

# General idea



# General idea





# Goal

To describe how students define explicitly some specific terms related to the concept of limit in contrast with their mathematical meaning.

## Conceptual Analysis of Specific Terms

*Conceptual Analysis of a term* is a procedure to identify uses and meanings of that term within a specific research in mathematics education or mathematics, in contrast with those from:

- researchers from the same field
- different scientific fields
- everyday life or own students and teachers

## Conceptual Analysis of Specific Terms

*Conceptual Analysis of a term* is a procedure to identify uses and meanings of that term within a specific research in mathematics education or mathematics, in contrast with those from:

- researchers from the same field
- different scientific fields
- everyday life or own students and teachers

## Conceptual Analysis of Specific Terms

*Conceptual Analysis of a term* is a procedure to identify uses and meanings of that term within a specific research in mathematics education or mathematics, in contrast with those from:

- researchers from the same field
- different scientific fields
- everyday life or own students and teachers

# Subjects

- 33 subjects were selected, from the previous study, a year later, who were taking the subject of Mathematics.
- The students were studying the second year of non-compulsory secondary education (Bachillerato), 17-18 years old.
- They were chosen deliberately and based on their availability.

# Subjects

- 33 subjects were selected, from the previous study, a year later, who were taking the subject of Mathematics.
- The students were studying the second year of non-compulsory secondary education (Bachillerato), 17-18 years old.
- They were chosen deliberately and based on their availability.

# Subjects

- 33 subjects were selected, from the previous study, a year later, who were taking the subject of Mathematics.
- The students were studying the second year of non-compulsory secondary education (Bachillerato), 17-18 years old.
- They were chosen deliberately and based on their availability.

# Instrument

- A semistructured interview was conducted in an ordinary classroom.
- The implementation protocol was a previous request to the students to write their answers on the answer sheet.
- A later discussion of responses that was audio recorded.
- The subjects were organised into nine groups with 3-5 components, in order to facilitate the interaction between the subjects and the researcher.



# Instrument

- A semistructured interview was conducted in an ordinary classroom.
- The implementation protocol was a previous request to the students to write their answers on the answer sheet.
- A later discussion of responses that was audio recorded.
- The subjects were organised into nine groups with 3-5 components, in order to facilitate the interaction between the subjects and the researcher.

# Instrument

- A semistructured interview was conducted in an ordinary classroom.
- The implementation protocol was a previous request to the students to write their answers on the answer sheet.
- A later discussion of responses that was audio recorded.
- The subjects were organised into nine groups with 3-5 components, in order to facilitate the interaction between the subjects and the researcher.

# Instrument

- A semistructured interview was conducted in an ordinary classroom.
- The implementation protocol was a previous request to the students to write their answers on the answer sheet.
- A later discussion of responses that was audio recorded.
- The subjects were organised into nine groups with 3-5 components, in order to facilitate the interaction between the subjects and the researcher.

## Pre-interview

This is the only question to answer, before discussion, for 15 minutes:

### Question

*Describe in each gap how you understand the following terms: “to approach,” “to tend,” “to reach,” “to exceed,” and “to converge” in the context of finite limit of a function at a point.*

## Pre-interview

This is the only question to answer, before discussion, for 15 minutes:

### Question

*Describe in each gap how you understand the following terms: “to approach,” “to tend,” “to reach,” “to exceed,” and “to converge” in the context of finite limit of a function at a point.*

# Interview

- In order to help the students to better express their ideas and to provoke new ones, the researcher included some examples or situations.
- For example, in the case of the terms “to reach,” “to exceed” and “to converge” we provided examples of graphs.

# Interview

- In order to help the students to better express their ideas and to provoke new ones, the researcher included some examples or situations.
- For example, in the case of the terms “to reach,” “to exceed” and “to converge” we provided examples of graphs.

## Analysis of discussion

From the analysis of discussion, the main thing is the existence of changes in the previous answers from students, both spontaneous and stimulated by the researcher.



# Differences of meaning between “to approach” and “to tend”

## Differences from discussion

- Reachability of the limit:

*I think that when a function tends can touch the number and when it approaches not. Because “to tend” can attain a concrete number, and “to approach” can be it is near that value.*

# Differences of meaning between “to approach” and “to tend”

## Differences from discussion

- Different behaviour with errors of approximation:

*The difference between 3 and 2.999999 [to tend] is less than that one between 4 and 2.999999 [to approach].*

# Differences of meaning between “to approach” and “to tend”

## Differences from discussion

- closeness versus orientation:

*“To approach” is as close as possible, and “to tend” is to go in a direction but it does not have to be so close.*

# Differences of meaning between “to approach” and “to tend”

## Differences from discussion

- Technical usage:

*I think that “to tend” is more suitable for  $x$ -values, and “to approach” would be the function, the image.*

# Differences of meaning between “to approach” and “to tend”

## Differences from discussion

- Intentionality:

*I think that “to approach” is intentioned, you want to go toward there, and “to tend” is at random.*

# New aspects of meaning of “to reach” and “to exceed”

## New aspects of meaning of “to reach”

- “to be reached” as different from “to be reachable”:

*If the limit is “reached,” I understand the same value, but at different points. But if the limit is “reachable,” I think about a “small function”. [“small function” is used to focus on a small neighbourhood of the point  $x=a$ ]*

# New aspects of meaning of “to reach” and “to exceed”

## New aspects of meaning of “to exceed”

- Function must be monotone :

*I think that it exceeds because it follows the same trajectory.*

# New aspects of meaning of “to reach” and “to exceed”

## New aspects of meaning of “to exceed”

- Local character of exceedability :

*I don't know. I think that it is not exceedable, because it would be a **relative maximum and at this point there aren't bigger values.***



## Conclusions

- C1. Most of definitions of the specific terms provided by students are coherent with their colloquial and everyday uses, except “to tend”, whose technical use in mathematics is relevant.

| Specific terms | Meanings                               | Frequencies (N=33) |
|----------------|--|--------------------|
| To Tend        | B1. To approach                        | 9                  |
|                | B1.1. Not to reach the limit           | 4                  |
|                | B1.2. To approach <i>more and more</i> | 3                  |
|                | B2. Technical usage                    | 8                  |
|                | B3. Subjective                         | 2                  |
|                | B4. Other/No answer                    | 7                  |

## Conclusions

- C2. Specific terms “to approach” and “to tend” were widely differentiated in written records (only 9 out of  $33-7=26$  valid answers considered these terms equivalent)

| Specific terms | Meanings                               | Frequencies (N=33) |
|----------------|--|--------------------|
| To Tend        | <b>B1. To approach</b>                 | <b>9</b>           |
|                | B1.1. Not to reach the limit           | 4                  |
|                | B1.2. To approach <i>more and more</i> | 3                  |
|                | B2. Technical usage                    | 8                  |
|                | B3. Subjective                         | 2                  |
|                | <b>B4. Other/No answer</b>             | <b>7</b>           |

## Conclusions

- C3. From discussion, students were able to establish other differences between “to approach” and “to tend”.
- C4. From discussion, students were able to explain better their conceptions about reachable limit and exceedable limit, in fact, their previous answers were very imprecise.

## Conclusions

- C3. From discussion, students were able to establish other differences between “to approach” and “to tend”.
- C4. From discussion, students were able to explain better their conceptions about reachable limit and exceedable limit, in fact, their previous answers were very imprecise.

## Conclusions

C5. The specific term “to converge” was at the beginning unknown for students, but by appropriate stimuli, students could provide alternative definitions.

| Specific terms | Meanings  | Frequencies (N=33) |
|----------------|---|--------------------|
|                | E1. The function is above the limit all the time                | 2                  |
|                | E2. The function is below the limit all the time                | 2                  |
|                | <b>E3. To tend</b>  | <b>1</b>           |
| To converge    | E4. To reach  | 1                  |
|                | <b>E5. The right and left-hand limits are the same</b>          | <b>3</b>           |
|                | E6. The function takes the same value than the limit (To reach) | 1                  |
|                | <b>E7. Two functions or straight lines intersect at a point</b> | <b>9</b>           |
|                | <b>E8. Other/No answer</b>                                      | <b>14</b>          |



## MEANINGS OF SPECIFIC TERMS RELATED TO THE CONCEPT OF FINITE LIMIT OF A FUNCTION AT ONE POINT

J.A. Fernández-Plaza, J.F. Ruiz-Hidalgo and L. Rico

Department of Didactics of Mathematics  
University of Granada-Spain

37<sup>th</sup> Conference of the International Group for the  
Psychology of Mathematics Education, Kiel, Germany  
July-August, 2013